

Probability-based estimate of tropical cyclone damage: An explicit approach and application to Hong Kong, China

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ABSTRACT

Tropical cyclones (TCs) are among the most costly natural hazards affecting civil construction in coastal areas, which have caused extensive economic losses and social disruption world-widely. Moreover, the intensity and/or frequency of future TCs have been projected to change with time due to the potential impacts of climate change, indicating the potential of ever larger TC damages for coastal regions. The estimate of future TC damage should be conducted under a probability-based framework, taking into account the uncertainties associated with both the TC characteristics (e.g., intensity and frequency) and the vulnerability of the coastal communities of interest. In this paper, an explicit approach is developed to evaluate the TC damage for TC-prone areas considering the potential impacts of climate change. The mean value and variance, as well as the cumulative density function of cumulative TC damage are estimated quantitatively. The proposed method is applied to TC damage assessment of Hong Kong, China – a TC-prone area which has suffered severely from historical cyclones. Sensitivity analysis is also performed to investigate the impacts of the time-variant characteristics of both the TC process and the TC damage conditional on one TC event on the cumulative damage costs for coastal areas.

1. Introduction

Tropical cyclone (TC) winds, rainfalls and storm surges are responsible for the major natural hazards in coastal areas, which having caused enormous economic losses and social disruption around the world. For example, in the United States, eight out of the ten most expensive catastrophes before 2006 in terms of insured loss had been triggered by TCs (known as hurricanes in the Atlantic Basin) [1]. In China, historical records show that totally 181 TCs (known as typhoons in the Pacific Basin) made landfall during the period 1980–2004, resulting in cumulative damage costs of 422.36 billion Chinese Yuan (CNY) [2]. Moreover, the population and wealth in coastal areas (most are TC-prone regions) have a considerable steady increase, indicating the potential of ever larger economic and social losses in the future. The increasing importance is then evident to advance building and structural practice through improving predictions of TC damage for civil constructions and thereby supporting implementation of strategies for enhancing the structural performance economically [3–5].

The future TCs have been projected to respond to climate change in many studies [6–9]. For instance, Mudd et al. [10] used a regression-based approach to show that in the Atlantic Basin, the annual occurrence rate of TCs (hurricanes) may increase from 8.4/year to 13.9/year at the end of the 21st century. Knutson et al. [11] predicted that the

averaged intensity of future TCs at a global scale will likely become stronger with an increase of 2–11% by year 2100 due to the potential impacts of greenhouse warming. However, there are also some arguments that the future TC intensity may increase while the frequency would decrease subjected to climate change (e.g., [12,13]). Emanuel [14] reported that despite the insignificant trend in future TC frequency, an upward trend in TC destructive potential can be observed. As a result, the time-variation of TC characteristics (e.g., intensity, frequency) should be well incorporated in terms of estimating the TC damage. Some previous studies [3,15] considered a stationary TC process with constant occurrence rate and time-invariant intensity (measured by the maximum TC wind speed) and thus failed to reflect the non-stationarity in TC process subjected to the potential impacts of climate change [16]. Later researches [17,18] took into account the roles of the time-variant TC characteristics in damage assessment, employing a probabilistic model of the ‘annual’ maximum TC wind speed to represent the TC wind risk. Such a probabilistic model, however, may lead to an overestimated TC damage in the presence of the intermittence of TCs in some regions. Li et al. [19] and Wang et al. [20–22] employed a Poisson stochastic process to model the TC occurrence and assessed the TC damage based on a vulnerability model as a function of the maximum TC wind speed. The mean value and variance of the cumulative damage have been considered in existing

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methods regarding TC damage assessment, which cannot fully reflect all the characteristics of the TC damage under a probability-based context (e.g., the upper tail behaviour or a characteristic value with a 90th percentile value). A Gamma distribution has been suggested to approximately fit the probability distribution of the TC damage [21,22]. However, the random nature associated with the TC damage has not been fully investigated. Moreover, in terms of damage assessment, especially for long-term reference periods, the utilization of a vulnerability model that was developed based on the damage reports/data associated with a single (or limited) TC event(s), as has been widely used in previous studies, would become questionable considering the time-variation of a community's vulnerability (e.g., enhanced structural design and practice), indicating the importance of developing a vulnerability model that can capture the change of a community's hazard-resisting capacity with time for use in TC damage assessment.

The scope of this paper is to develop a probabilistic approach for TC damage assessment for TC-prone areas under the context of climate change, taking into consideration the non-stationarity in the TC process. This paper is organized as follows. Section 2 presents probabilistic models for both the stochastic TC occurrence process and TC damage. Section 3 develops a moment-based approach for TC damage assessment, which gives a straightforward description of the magnitude and variation of the TC damage. In Section 4, the probability distribution of cumulative TC damage is developed in an explicit form. Illustrative analyses are conducted in Section 5 to demonstrate the applicability of the proposed method, choosing Hong Kong, China, as an example. The conclusions are finally formulated in Section 6.

2. Probabilistic models of TC process and TC damage

2.1. TC occurrence model

For a specific region of interest, TC events occur randomly in time. A Poisson point process can be used to describe the TC occurrence process [19–24]. With a mean occurrence rate of λ , the probability that k events occur within a time interval of $(0, T]$ is determined by

$$\Pr(N_T = k) = \frac{(\lambda T)^k}{k!} \exp(-\lambda T), \quad k = 0, 1, 2, \dots \quad (1)$$

in which $\Pr(\cdot)$ denotes the probability of the event in the bracket, and N_T is the number of events within $(0, T]$. Wang et al. [21] examined the historical TC data for both the Miami-Dade County (Florida, USA) and the Florida State respectively. For both scales of region, they found that a Poisson point process can reasonably model the occurrence process of TCs that made landfall. Herein, we examine the distribution of annual number of TCs that have caused direct economic losses to Hong Kong, China – a region that has suffered severely from historical cyclones. The probability mass function of the annual TC numbers is plotted in Fig. 1,

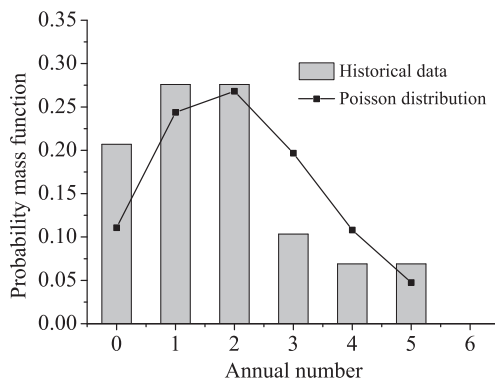


Fig. 1. Probability mass function of the annual number of TCs that caused economic losses to Hong Kong.

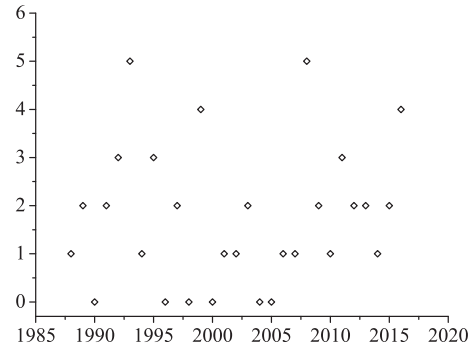


Fig. 2. Annual number of TCs that caused economic losses to Hong Kong: from 1988 to 2016.

where a historical period of 1988–2016 is covered. The data are available from the annual reports on TCs released by the Hong Kong Observatory of HKSAR (<http://gb.weather.gov.hk/publica/pubtcc.htm>) [25]. Totally 51 TC events had triggered direct damage losses within the considered 29 years, yielding an annual occurrence rate of 1.76/year. Fig. 1 shows that the annual number can be well modeled by a Poisson distribution with $\lambda = 2.2/\text{year}$ (c.f. Eq. (1))¹ at a significance level of 20%.

Further, taking into account the potential impacts of climate change, the occurrence rate of future TCs may vary with time correspondingly. In such a case, a non-homogeneous Poisson point process can be used to describe the non-stationarity in the TC occurrence. With a time-variant occurrence rate of $\lambda(t)$, Eq. (1) is rewritten as follows,

$$\Pr(N_T = k) = \frac{(\int_0^T \lambda(t) dt)^k \cdot \exp(-\int_0^T \lambda(t) dt)}{k!}, \quad k = 0, 1, 2, \dots \quad (2)$$

For illustration purpose, we examine the annual numbers of historical TCs that affected Hong Kong, as have been reported in [25]. Fig. 2 presents the annual number of TCs that resulted in direct economic losses to Hong Kong within a time period of 1988–2016, where a slightly upward trend can be observed, with a R-square value of 0.011. This trend is not strongly evident of the non-stationarity in the TC occurrence from a view of statistics, due to the relatively small sample size and short duration that have been considered. The non-stationarity in the TC occurrence process will, however, be discussed in the following to illustratively investigate its role in TC damage assessment, taking into account the potential changing scenarios of TCs in the future, as observed elsewhere [13].

2.2. TC damage model

The TC-induced damage for a specific region of interest associated with one TC event is by nature random, with uncertainties arising from both the TC intensity and the hazard-resisting capacity of the area. One of the methods to quantitatively measure the TC damage, under a probability-based framework, is to use a vulnerability model which links the possibility of occurrence of certain levels of damage to the maximum TC wind speed [3,26–28]. For instance, Stewart et al. [3] used the claim and loss information from Hurricane Hugo (1989) and Hurricane Andrew (1992) that were obtained from an insurer and developed a damage model as a function of the 10-min mean surface wind speed, where the ‘damage’ was defined as the amount paid out by the insurer divided by the total insured value. This regression-based vulnerability model was further used in damage assessment of coastal

¹ While the mean value of the samples is 1.76/year in Fig. 1, due to the dependence of the mean value and variance of a Poisson distribution (both equal to λ), $\lambda = 2.2/\text{year}$ instead of 1.76/year is found to maximize the p-value (0.5053) from a view of goodness-of-fit test. This observation suggests that the TC risk may be underestimated if one simply set λ in Eq. (1) as the mean value of the samples.

areas considering the potential impacts of climate change in later studies [17–22]. Using this model, the time-variation in the TC damage is reflected by either the non-stationarity in the TC process (e.g., time-variant intensity and/or frequency) or a simple modification of the vulnerability model (e.g., applying a deterministic reduction factor to the overall damage, as done in [17]). Wang et al. [22] further discussed the impact of vulnerability model uncertainty on the cumulative damage assessment. However, the regression-based models have been, for the most part, developed based on the claim data that are available from insurance companies and may not span all significant wind speeds, especially those extreme ones with large return periods [29]. Moreover, the use of a single vulnerability model (or the simply modified ones) in the damage assessment of a long-term period is questionable as it cannot reflect the improvement of the hazard-resisting capacity of the area (e.g., the improvement of building code or practice). An illustrative example is that after Hurricane Andrew (1992), the Florida Building Code required significant changes to the construction of civil structures with greater resilience [30,31]. As such, a more accurate damage model that reflects the time-variation of the area's hazard-resisting capacity should be used in TC damage assessment, especially for long-term reference periods. Some other remarkable methods to establish a vulnerability model for a specific region of interest include the engineering-based damage assessment by considering sophisticated structure–load interaction and structural resistance, and the assembly-based vulnerability method linking the engineering demand variable to the response of structural and nonstructural systems and building contents damage [15,29,32–34]. However, the disadvantage of these methods is that they require multidisciplinary modelling skills, validation ensured by realistic TC damage data, and are of significant complexity compared with regression-based models, calling for more significant research efforts for the application to practical engineering [22,35]. As a result, this paper only discusses the regression-based vulnerability models.

The probabilistic model of TC damage is expected to be developed based on the damage loss records incorporating all the significant items, as soon as the historical data are accessible. Taking Hong Kong, China as an example, the TC damage losses associated with each TC event are available from Hong Kong Observatory (in the form of Annual Reports on TCs). The damage losses includes those of agriculture, public works facilities, public utilities, private property and industry [25]. Fig. 3(a) shows the TC damage losses associated with each TC event within a reference period of 1988–2016. Two significant cyclones, Typhoon Dot (1993), and Typhoon Sam (1999), led to severe damage losses of 128.2 and 131.51 million HK\$, respectively (as of the year of valuation). Furthermore, taking into account the time value of money, the present values of the damage losses in Fig. 3(a) are reconsidered in Fig. 3(b) (as of year 2016), where a discount rate of 3.5% is adopted, as suggested in [36].

As has been discussed before, many existing vulnerability models for TC damage assessment have been developed as a function of the TC wind speed [3,26–28]. Examining the case for Hong Kong, Fig. 4 reviews the TC events in Fig. 3 and presents the TC damage costs and the maximum gust wind speed associated with each historical TC event. A discount rate of 3.5% is considered in Fig. 4(b). For both cases in Fig. 4, the linear correlation coefficients between the TC damage costs and the maximum gust wind speed are weak (0.2778 and 0.2521, respectively), indicating that the damage level associated with a TC event cannot be fully determined via considering the maximum wind speed only – that is, the association between the TC damage and the wind speed cannot be well described using a single functional relationship. This is explained by the fact that the TC damage is not only affected by the maximum wind speed but also some other factors such as the TC track and the TC decay process [37,38], as well as some ripple effects associated with the infrastructure systems within the area. Some previous works have also examined the historical data by considering the relationship between the damage with more than one factors of the TCs rather than the maximum wind speed only (e.g., translational track, size and duration) [14,39–41]. However, those empirical functional relationships are nevertheless associated with significant uncertainties due to the fact that only the TC profiles have been considered while some other elements such as the building vulnerability, terrain conditions and the non-structural damages remain unaddressed.

The probability distribution of the TC damage costs associated with each TC event can be fitted directly based on the historically recorded data. Such a fitting-based model can make direct use of the historical information and thus gives a “best” estimate of the probability distribution of the TC damage. Moreover, the time-variation of both the TC temporal characteristics (e.g., frequency and intensity) and the area's hazard-resisting capacity would be automatically reflected by this fitting-based model because the damage data that have been used cover a historical period rather than a specific time point. For comparison purpose, recall that the vulnerability model in [3,23] has been developed based on historical TCs (hurricanes) Hugo (1989) and Andrew (1992) only, and thus does not involve the information on the subsequent changes in the building code and construction practice for the built environment. It is nevertheless noticed that the disadvantage of the fitting-based damage model is that it does not incorporate the relationship between the damage and the key factors such as the maximum wind speed, and thus cannot be used to identify the role of TC characteristics in the TC damage, especially in the presence of the non-stationarity in the TC processes due to the potential impacts of climate change. Finally, the historical TC damage data examined herein cover a historical period of 1988–2016, which are assumed in this paper to well predict the “future” changes after year 2016. Fig. 5 presents the histogram of the damage records (as in Fig. 3(b)) and the probability density function (PDF) of a Gamma distribution to fit the samples. If a

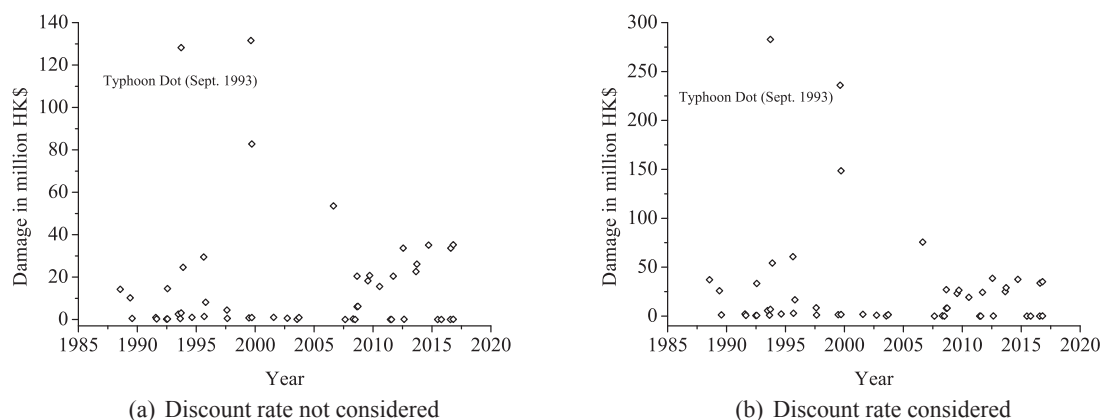


Fig. 3. Time-variant trend of TC damage associated with each TC event.

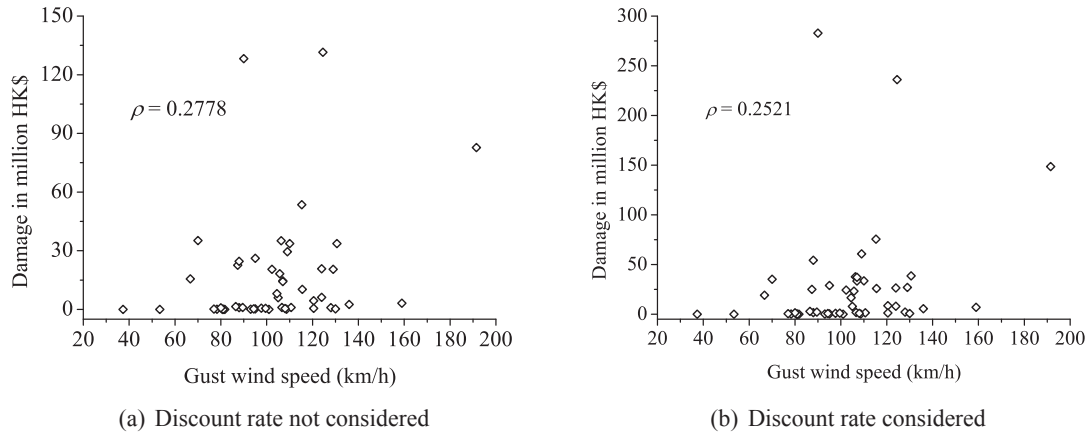


Fig. 4. No strong correlation observed from the relationship between TC damage costs and the maximum gust wind speed.

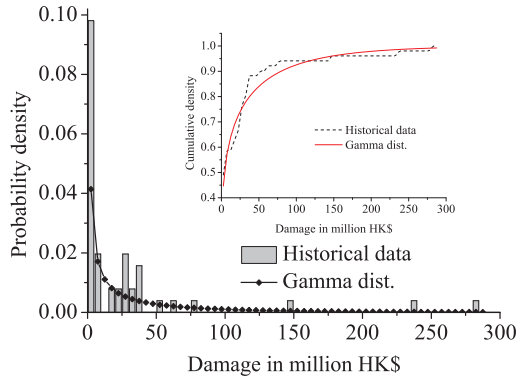


Fig. 5. The histogram of the damage records and the PDF of a Gamma distribution to fit the samples.

random variable, say, X , follows a Gamma distribution, then its PDF, $f_X(x)$, takes the form of

$$f_X(x) = \frac{(x/\beta)^{\alpha-1}}{\beta\Gamma(\alpha)} \exp(-x/\beta), \quad x \in [0, +\infty) \quad (3)$$

where α and β are the shape parameter and rate parameter, respectively, and $\Gamma(\cdot)$ is the Gamma function, $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \exp(-x) dx$. With Eq. (3), the mean value and variance of X are $\alpha\beta$ and $\alpha\beta^2$, respectively, giving a coefficient of variation (COV) of $1/\sqrt{\alpha}$. Fitting analysis in Fig. 5 gives $\alpha = 0.2361$, and $\beta = 111.94$, which results in a p-value of 0.6177 and is accepted at a significance level of 20%. It is emphasized that the damage losses associated with Typhoons Dot and Sam (c.f. Fig. 3) represent two extreme cases of severe damage, and thus cannot be simply treated as outliers [42,43] in terms of statistics-based fitting of the damage distribution; in fact, the exclusion of the two extreme cases will unavoidably lead to an underestimated TC damage.

3. Moment-based estimate of TC damage

For a coastal region of interest, during the time interval $(0, T]$, the cumulative TC damage costs, $D(T)$, is estimated by $D(T) = \sum_{j=1}^{N_T} D_j$, where N_T is the number of TCs within $(0, T]$ that cause economic losses to the region, and D_j is the damage costs associated with the j th TC event. The estimate of $D(T)$ is based on the assumption that the damaged buildings/structures are restored to the pre-damage state before the occurrence of the next TC event [21], which is likely to result in a relatively conservative estimate. In this section, the mean value and variance of $D(T)$ are evaluated, which provide a straightforward description of the magnitude and variation of the cumulative damage costs. With the TC occurrence process modeled as a non-stationary

Poisson process with a time-variant occurrence rate of $\lambda(t)$, we divide the time period $(0, T]$ into N identical sections (N is large enough so that at most one TC event may occur during each time interval). With this, the possibility of occurrence of a TC event during the j th time interval is $\lambda(\frac{jT}{N}) \cdot \frac{T}{N}$. By definition, it follows,

$$D(T) = \sum_{j=1}^N \hat{D}_j \quad (4)$$

where $\hat{D}_j$ is the hurricane damage costs associated with the j th time interval. Defining $t_j = jT/N$, one has

$$\hat{D}_j = \frac{B_j \cdot \tilde{D}(t_j)}{[1 + r(t_j)]^{t_j}} \quad (5)$$

in which $\tilde{D}(t_j)$ is the TC damage given the occurrence of one TC event in the j th time interval, $r(t)$ is the time-variant discount rate which enables the future TC damage costs to be valued in present terms [44], and B_j is a Bernoulli random variable,

$$\Pr(B_j = 1) = \lambda(t_j) \cdot \frac{T}{N}; \Pr(B_j = 0) = 1 - \Pr(B_j = 1) \quad (6)$$

We further define a variable $D(t_j) = \frac{\tilde{D}(t_j)}{[1 + r(t_j)]^{t_j}}$ for each j , with which Eq. (5) simply becomes $\hat{D}_j = B_j D(t_j)$. Now, with Eq. (4),

$$\mathbb{E}[D(T)] = \sum_{j=1}^N \mathbb{E}(\hat{D}_j) = \sum_{j=1}^N \lambda(t_j) \cdot \frac{T}{N} \cdot \frac{\mathbb{E}(\tilde{D}_j)}{[1 + r(t_j)]^{t_j}} \quad (7)$$

where $\mathbb{E}(\cdot)$ denotes the mean value of the variable in the bracket. If the future TC damage associated with a TC event (i.e., $\tilde{D}(t_j)$) is known to have a mean value of $\mu_D(t)$ and a standard deviation of $\sigma_D(t)$ at time t , Eq. (7) becomes

$$\mathbb{E}[D(T)] = \int_0^T \frac{\lambda(t) \mu_D(t)}{[1 + r(t)]^{t}} dt \quad (8)$$

Similarly,

$$\begin{aligned} \mathbb{V}[D(T)] &= \sum_{j=1}^N \mathbb{V}(\hat{D}_j) = \sum_{j=1}^N \lambda(t_j) \cdot \frac{T}{N} \cdot \frac{\mathbb{E}(\tilde{D}_j^2)}{[1 + r(t_j)]^{2t_j}} \\ &= \int_0^T \frac{\lambda(t) [\mu_D^2(t) + \sigma_D^2(t)]}{[1 + r(t)]^{2t}} dt \end{aligned} \quad (9)$$

where $\mathbb{V}(\cdot)$ denotes the variance of the variable in the bracket.

Eqs. (8) and (9) are the proposed method to assess the cumulative TC damage costs in the presence of both the non-stationarity of TCs (reflected by the time-variant characteristics of intensity and/or frequency) and the time-variation of vulnerability model. The probabilistic behaviour of cumulative TC damage can be assessed by assuming an approximate distribution type for $D(T)$, as has been done in previous

works (e.g., a Gamma distribution as suggested by Wang et al. [21,22]). In an attempt to achieve a further insight into the probabilistic behaviour of the cumulative damage, in addition to the mean value and variance (c.f. Eqs. (8) and (9)), the accurate probability distribution of $D(T)$ is considered, as will be discussed in Section 4.

4. Probability distribution of TC damage

4.1. General formulation

Statistical parameters such as the mean value fail to reflect all aspects of TC damage costs with the probability distribution remaining unaddressed. In this section, the distribution of $D(T)$ is developed with the help of moment generating function and the characteristic function [45–47]. The moment generating function, $\phi_X(\tau)$, of a random variable X is defined for all real values τ by

$$\phi_X(t) = \mathbb{E}[\exp(X\tau)] \quad (10)$$

An important property of moment generating functions is that the moment generating function of the sum of independent random variables equals the product of the individual moment generating functions. For example, consider n independent random variables $X_1, X_2, \dots, X_n$ with moment generating functions of $\phi_1(\tau), \phi_2(\tau), \dots, \phi_n(\tau)$, respectively. The sum of the n variables, $X = X_1 + X_2 + \dots + X_n$, has a moment generating function of $\prod_{k=1}^n \phi_k(\tau)$.

Now we consider the distribution of cumulative hurricane damage costs, $D(T)$, during time period $(0, T]$. With Eq. (4), the moment generating function of $D(T), \phi_D(\tau)$, is obtained as

$$\phi_D(\tau) = \mathbb{E}\left[\exp\left(\sum_{k=1}^N \tau \widehat{D}_k\right)\right] \quad (11)$$

By noting the independence² between $\widehat{D}_1, \widehat{D}_2, \dots, \widehat{D}_N$ and the Taylor expansion of an exponential function (i.e., $\exp(x) = \sum_{j=0}^{\infty} \frac{x^j}{j!}$ holds for any number x), Eq. (11) becomes

$$\begin{aligned} \phi_D(\tau) &= \mathbb{E}\left[\prod_{k=1}^N \exp(\tau \widehat{D}_k)\right] = \mathbb{E}\left[\prod_{k=1}^N \left[\sum_{j=0}^{\infty} \frac{(\tau \widehat{D}_k)^j}{j!}\right]\right] \\ &= \prod_{k=1}^N \left[\sum_{j=0}^{\infty} \frac{\mathbb{E}(\tau \widehat{D}_k)^j}{j!}\right] \end{aligned} \quad (12)$$

Taking the logarithmic form for both sides of Eq. (12), since $\ln(1+x)$ approximates x for a small value of x , we have

$$\begin{aligned} \phi_D(\tau) &= \exp\left\{\sum_{k=1}^N \ln\left[\sum_{j=0}^{\infty} \frac{\mathbb{E}(\tau \widehat{D}_k)^j}{j!}\right]\right\} = \exp\left\{\sum_{k=1}^N \sum_{j=1}^{\infty} \frac{\mathbb{E}(\tau \widehat{D}_k)^j}{j!}\right\} \\ &= \exp\left(\sum_{j=1}^{\infty} \xi_j \tau^j\right) \end{aligned} \quad (13)$$

where

$$\xi_j = \frac{1}{j!} \sum_{k=1}^N \mathbb{E}[(\widehat{D}_k)^j] \quad (14)$$

The characteristic function, $\phi_{iX}(it)$, of a random variable X is introduced to link the probability density function and the moment generating function of X ; it is actually the Fourier transformation of the PDF of X . $\phi_{iX}(i\tau)$ is defined for all real values τ by

$$\phi_{iX}(i\tau) = \mathbb{E}(e^{iX\tau}) = \int_{-\infty}^{\infty} \exp(i\tau x) \cdot dF_X(x) \quad (15)$$

where $F_X(x)$ is the cumulative density function (CDF) of X ; $i = \sqrt{-1}$. If X is a continuous random variable with a PDF of $f_X(x)$, Eq. (15) becomes

$$\phi_{iX}(i\tau) = \int_{-\infty}^{\infty} \exp(i\tau x) \cdot f_X(x) dx \quad (16)$$

Eq. (16) implies that $\phi_X(\tau)$ and $f_X(x)$ is a Fourier transform pair. Thus,

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-ix\tau) \phi_{iX}(i\tau) d\tau \quad (17)$$

Substituting Eq. (13) into Eq. (17), the PDF of $D(T), f_D(x)$, is obtained as follows,

$$\begin{aligned} f_D(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m}\right) \cdot \exp\left(i \sum_{m=0}^{\infty} (-1)^m \xi_{2m+1} \tau^{2m+1} - ix\tau\right) d\tau \end{aligned} \quad (18)$$

By noting that for any real number $x, \exp(ix) = \cos x + i \sin x$, we let

$$\theta(\tau, x) = \sum_{m=0}^{\infty} (-1)^m \xi_{2m+1} \tau^{2m+1} - x\tau \quad (19)$$

with which Eq. (18) becomes

$$\begin{aligned} f_D(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m}\right) \cos\theta(\tau, x) d\tau \\ &\quad + i \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m}\right) \sin\theta(\tau, x) d\tau \end{aligned} \quad (20)$$

Since the term $\exp(\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m}) \cdot \sin\theta(\tau, x)$ in Eq. (20) is an odd function of τ , the following equation holds,

$$\int_{-\infty}^{\infty} \exp\left(\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m}\right) \cdot \sin\theta(\tau, x) d\tau = 0 \quad (21)$$

Thus, Eq. (20) becomes

$$f_D(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m}\right) \cdot \cos\theta(\tau, x) d\tau \quad (22)$$

Recall Eq. (14), with which one has

$$\begin{aligned} \xi_j &= \lim_{N \rightarrow \infty} \frac{1}{j!} \sum_{k=1}^N \mathbb{E}(B_k D^j(t_k)) = \lim_{N \rightarrow \infty} \frac{1}{j!} \sum_{k=1}^N \mathbb{E}[(D(t_k))^j] \cdot \lambda(t_k) \cdot \frac{T}{N} \\ &= \lim_{N \rightarrow \infty} \frac{1}{j!} \sum_{k=1}^N \frac{\mathbb{E}[(\widetilde{D}(t_k))^j]}{(1+r(t_k))^{jk}} \cdot \lambda(t_k) \cdot \frac{T}{N} \end{aligned} \quad (23)$$

By definition,

$$\mathbb{E}[(\widetilde{D}(t_k))^j] = \int_0^{\infty} z^j f_0(z, t_k) dz \quad (24)$$

where $f_0(z, t_k)$ is the PDF of $\widetilde{D}(t_k)$. Substituting Eq. (24) into Eq. (23), we have

$$\xi_j = \frac{1}{j!} \int_0^T \int_0^{\infty} \left(\frac{z}{(1+r(t))^t}\right)^j \lambda(t) f_0(z, t) dz dt \quad (25)$$

With this,

$$\xi_{2m} = \frac{1}{(2m)!} \int_0^T \int_0^{\infty} \left(\frac{z}{(1+r(t))^t}\right)^{2m} \lambda(t) f_0(z, t) dz dt \quad (26)$$

and further,

$$\sum_{m=1}^{\infty} (-1)^m \xi_{2m} \tau^{2m} = \int_0^T \int_0^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{z\tau}{(1+r(t))^t}\right)^{2m} \cdot \lambda(t) f_0(z, t) dz dt \quad (27)$$

² The statistical independence between $\widehat{D}_1, \widehat{D}_2, \dots, \widehat{D}_N$ is due to the assumptions that (a) the TC occurrence is a Poisson process; and (b) the damaged buildings/structures are restored to the pre-damage state before the occurrence of the next TC event [21], as aforementioned. Assumption (a) has been widely used in the literature to model the TC occurrence process [19–24]. One can refer to Refs. [19,21] for the impacts of the temporal correlation in the TC process on the TC damage.

We let $\delta(\tau) = \sum_{m=1}^{\infty} (-1)^m \frac{\tau^{2m}}{(2m)!}$ for simplicity. By noting that $\cos x = 1 + \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} x^{2m}$ for any real number x , Eq. (27) becomes

$$\delta(\tau) = \int_0^T \int_0^{\infty} \cos\left(\frac{z\tau}{(1+r(t))^t}\right) \cdot \lambda(t) f_0(z, t) dz dt - \int_0^T \lambda(t) dt \quad (28)$$

Similarly, since $\sin x = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)!} x^{2m+1}$ holds for any real number x , the item $\theta(\tau, x)$ in Eq. (19) becomes

$$\theta(\tau, x) = \int_0^T \int_0^{\infty} \sin\left(\frac{z\tau}{(1+r(t))^t}\right) \lambda(t) f_0(z, t) dz dt - x\tau \quad (29)$$

Substituting Eqs. (28) and (29) into Eq. (22), it follows,

$$f_D(x) = \frac{1}{\pi} \int_0^{\infty} \exp(\delta(\tau)) \cdot \cos\theta(\tau, x) d\tau = \frac{1}{\pi} \int_0^{\infty} \exp(\delta(\tau)) \cdot \cos[\theta_0(\tau) - x\tau] d\tau \quad (30)$$

where $\theta_0(\tau) = \theta(\tau, x) + x\tau$. Eq. (30) presents an explicit form of the PDF of $D(T)$. Further, the CDF of $D(T)$, $F_D(x)$, is obtained as

$$F_D(x) = \int_0^x f_D(y) dy = \frac{1}{\pi} \int_0^{\infty} \frac{\exp(\delta(\tau))}{\tau} \cdot [\sin\theta_0(\tau) - \sin(\theta_0(\tau) - x\tau)] d\tau \quad (31)$$

The closed-form solutions of the probability distribution of $D(T)$ as in Eqs. (30) and (31) can be solved using numerical integral techniques. It is noticed that a twofold integral is included in the calculation of $f_D(x)$ and $F_D(x)$, if one first calculates $\theta(\tau)$ and $\delta(\tau)$ and stores them on disk for further use. Some numerical techniques such as the Simpson's rule [48] can be used to improve the integration efficiency. Although the Monte Carlo simulation (MCS) methods can also be used to find (usually approximate) the probability distribution of $D(T)$, it is argued that the closed-form analytical solutions can offer insights that may be difficult to achieve through MCS, may support and even accelerate the development of more complex simulation-based analyses [49]. Nevertheless, the simulation-based method can be used to verify the accuracy of the proposed explicit solutions.

4.2. Application to TC damage assessment: A simplified approach

The probability distribution of cumulative TC damage can be obtained with either Eq. (30) or (31), in which the calculation of both $\theta(\tau)$ and $\delta(\tau)$ (c.f. Eq. (19) and (28)) can be further simplified if assuming a Gamma distribution for the TC damage costs associated with each TC event, as will be discussed in this section.

First, note that for a random variable X following a Gamma distribution, if its PDF, $f_X(x)$, is expressed by Eq. (3), then the moment generating function is $\phi_X(\tau) = \left(\frac{1}{1-\beta\tau}\right)^\alpha$. With this,

$$\begin{aligned} \int_0^{\infty} \cos x \cdot f_X(x) dx + i \int_0^{\infty} \sin x \cdot f_X(x) dx &= \int_0^{\infty} \exp(ix) \cdot f_X(x) dx \\ &= \mathbb{E}[\exp(iX)] = \phi_{iX}(i) = \left(\frac{1}{1-\beta i}\right)^\alpha \\ &= \left(\frac{1+\beta i}{1+\beta^2}\right)^\alpha = \frac{\exp\left(i\alpha \cdot \arccos \frac{1}{\sqrt{1+\beta^2}}\right)}{(1+\beta^2)^{\alpha/2}} \\ &= \frac{\cos\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+\beta^2}}\right)}{(1+\beta^2)^{\alpha/2}} \\ &\quad + i \frac{\sin\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+\beta^2}}\right)}{(1+\beta^2)^{\alpha/2}} \end{aligned} \quad (32)$$

Comparing both sides of the equation, it follows,

$$\int_0^{\infty} \cos x \cdot f_X(x) dx = \frac{\cos\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+\beta^2}}\right)}{(1+\beta^2)^{\alpha/2}} \quad (33)$$

and

$$\int_0^{\infty} \sin x \cdot f_X(x) dx = \frac{\sin\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+\beta^2}}\right)}{(1+\beta^2)^{\alpha/2}} \quad (34)$$

A generalized form of Eq. (33) is given by

$$\begin{aligned} \int_0^{\infty} \cos(kx) \cdot f_X(x) dx &= \int_0^{\infty} \cos(kx) \frac{(x/\beta)^{\alpha-1} \exp(-x/\beta)}{\beta \Gamma(\alpha)} dx \\ y = kx &\Rightarrow \int_0^{\infty} \cos y \frac{y^{\alpha-1}}{(k\beta)^{\alpha} \Gamma(\alpha)} \exp(-y/(k\beta)) dy = \frac{\cos\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+(k\beta)^2}}\right)}{[1+(k\beta)^2]^{\alpha/2}} \end{aligned} \quad (35)$$

where k is an arbitrary positive constant. Similarly,

$$\int_0^{\infty} \sin(kx) \cdot f_X(x) dx = \frac{\sin\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+(k\beta)^2}}\right)}{[1+(k\beta)^2]^{\alpha/2}} \quad (36)$$

Thus, assigning $k = \frac{\tau}{(1+r(t))^t}$, the calculation of $\theta(\tau)$ and $\delta(\tau)$ in Eqs. (19) and (28) are simplified as,

$$\begin{aligned} \delta(\tau) &= \int_0^T \int_0^{\infty} \cos\left(\frac{z\tau}{(1+r(t))^t}\right) \cdot \lambda(t) f_0(z, t) dz dt - \int_0^T \lambda(t) dt \\ &= \int_0^T \lambda(t) \left[\int_0^{\infty} \cos\left(\frac{z\tau}{(1+r(t))^t}\right) \cdot f_0(z, t) dz \right] dt - \int_0^T \lambda(t) dt \\ &= \int_0^T \frac{\lambda(t) \cos\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+\left(\frac{\tau\beta(t)}{(1+r(t))^t}\right)^2}}\right)}{\left[1+\left(\frac{\tau\beta(t)}{(1+r(t))^t}\right)^2\right]^{\alpha/2}} dt - \int_0^T \lambda(t) dt \end{aligned} \quad (37)$$

Similarly,

$$\begin{aligned} \theta(\tau, x) &= \int_0^T \int_0^{\infty} \sin\left(\frac{z\tau}{(1+r(t))^t}\right) \lambda(t) f_0(z, t) dz dt - x\tau \\ &= \int_0^T \frac{\lambda(t) \sin\left(\alpha \cdot \arccos \frac{1}{\sqrt{1+\left(\frac{\tau\beta(t)}{(1+r(t))^t}\right)^2}}\right)}{\left[1+\left(\frac{\tau\beta(t)}{(1+r(t))^t}\right)^2\right]^{\alpha/2}} dt - x\tau \end{aligned} \quad (38)$$

It can be seen that only a onefold integral is involved in Eqs. (37) and (38), which will accelerate the calculation efficiency significantly compared with the calculation of a double-fold integral. Further, it is noticed that the calculation of the CDF of cumulative TC damage, $F_D(x)$ in Eq. (31), involves an improper integral, which is valid only if it is convergent. The convergence of F_D is discussed in Appendix A in detail. Moreover, for a more generalized case, the calculation of both $\theta(\tau)$ and $\delta(\tau)$ can also be simplified to a onefold integral for some other distributions of the TC damage costs. Illustratively, Appendix B discusses the simplification of $\theta(\tau)$ and $\delta(\tau)$ when the TC damage conditional on the occurrence of one TC event follows a normal distribution.

5. TC damage assessment: A case study in Hong Kong, China

Choosing Hong Kong, China as an example, the estimate of future TC damage costs is performed in this section using the proposed method. Hong Kong is a coastal area which has suffered significantly from severe TCs. For instance, during a historical period of 1956–2016, on average fifteen TCs fell within Hong Kong's area of responsibility annually [25]. Triggered by these TC events, the averaged economic losses valued 15.92 million HK\$ for each year (discount rate not considered).

5.1. Changing patterns of TC frequency and TC damage costs

The future TC occurrence rate and intensity may change with time

due to the potential impacts of climate change. While this non-stationarity in the TC process has been well acknowledged and widely discussed, yet it is still in debate how exactly the future changes will be. From a view of civil engineers, however, our focus is not on the mechanism-based prediction of future TC scenarios, but on the impacts of such time-variant changes on the estimate of damages that have been caused to coastal communities.

In order to demonstrate the applicability of the proposed method, the TC damage assessment is conducted choosing Hong Kong, China as the area of interest. The future changing scenarios of both TC frequency and TC damage costs associated with one successful TC event can be roughly estimated through analyzing historical records. First, in terms of future TC frequency, Fig. 1 shows that for Hong Kong area, the average annual number of TCs that have caused economic losses is estimated to be 2.2/year. This figure is slightly greater than that predicted from regression-based analysis (i.e., 1.96/year as of 2016 according to Fig. 2) and thus offers a relatively conservative estimate of the TC occurrence rate. Next, the TC damage associated with each successful TC event (discount rate considered) is modeled by a Gamma distribution. As shown in Fig. 5, the mean value and COV are 26.429 (in million HK\$) and 2.058, respectively, without the consideration of time-variation. This mean value is significantly greater than that estimated from Fig. 3(b) using a regression-based approach (16.67 million HK\$, at year 2016). In the following analyses, the initial time is set as year 2016. The initial TC occurrence rate is set 2.2/year and the initial mean value of TC damage costs is 16.67 (in million HK\$). The COV of TC damage is constant over time (2.058, corresponding to $\alpha = 0.2361$ in Eq. (3)), and a discount rate of 3.5% [36] is considered unless otherwise stated.

Taking into account the potential impacts of climate change, we consider the case where the TC occurrence rate changes linearly with time, and the TC damage associated with a successful TC event follows a Gamma distribution with a linearly changing mean value and a constant COV. Mathematically, it follows,

$$\lambda(t) = \lambda_0(1 + \kappa_\lambda t) \quad (39)$$

and

$$\beta(t) = \beta_0(1 + \kappa_\beta t) \quad (40)$$

where λ_0 and β_0 are the initial occurrence rate of TC events and the initial rate parameter of the Gamma-distributed TC damage (see Eq. (3)), and note that the mean value is proportional to β , κ_λ and κ_β are two time-invariant parameters reflecting the changing rates of TC frequency and mean intensity.

The changing rate of future TC frequency with time can be inferred from Fig. 2, where κ_λ is found to be 0.008. Similarly, the change rate of future TC damage costs, κ_β , is 0.021 as revealed in Fig. 3(a). Note that $\kappa_\lambda = 0.008$ so that the TC frequency would increase by 80% over 100 years and $\kappa_\beta = 0.021$ means that the mean value of TC damage costs associated with one successful TC event increases by 210% over a reference period of 100 years³. It is emphasized that the changing rates adopted here are only representative of a likely pattern in the future. Keeping this fact in mind, in order to investigate the effects of time-variant TC frequency and the damage magnitude (conditional on the occurrence of one TC event) on the cumulative damage costs, four additional changing patterns are considered, named Cases 1 through 4, as summarized in Table 1. The additional cases take into consideration

³ In order to test the applicability of the proposed method, a hind-casting analysis is conducted herein. We go back to year 2003 and use the “historical” data of 1988–2003 to predict the “future” TC damage over a period of 2004–2016. With this, according to Eqs. (8) and (9), the mean value and standard deviation of $D(13)$ (cumulative discounted TC damage losses within a period of 2004–2016) are estimated as 413.73 and 229.30 million HK\$, respectively. The historical “realistic” cumulative TC damage losses for this period is 263.16 million HK\$, which falls within the range of the predicted “mean $\pm$ standard deviation”, implying that the proposed method provides a good estimate of the future TC damage.

Table 1

Changing scenarios of TC frequency and TC damage costs associated with one successful TC event.

Case No.	κ_λ	κ_β
1 ^a	0	0
2	0.016	0
3	0	0.042
4	0.016	0.042
5 ^b	0.008	0.021

^a Case 1 corresponds to the case of a stationary TC process without consideration of the potential impacts of climate change and time-invariant vulnerability of the area.

^b Case 5 corresponds to the scenario of changing TC frequency and TC damage costs inferred from Figs. 2 and 3(a), respectively.

different potential scenarios in the TC frequency and the TC damage costs conditional on the occurrence of a TC event. The consideration of cases 1 through 5 in the following analyses is to investigate the impacts of potential changing scenarios of future TC events on the estimate of TC damage; the selection of a specific changing scenario of future TCs to support a decision-making finally falls within the decision-maker's side, which may need further evidence from the meteorologists.

5.2. Moment-based damage costs assessment

Fig. 6 plots the mean value and COV of cumulative TC damage costs, $D(T)$, for periods up to 100 years associated with the five changing patterns of future TCs as summarized in Table 1. It is seen that the expected cumulative damage costs increase as the reference period becomes longer, which is characteristic of the cumulation of damage. The mean value of damage costs associated with case 4 is highest, followed by those associated with cases 3, 5, 2 and 1 in Fig. 6(a), indicating that the increase of TC frequency and TC severity (reflected by the damage costs associated with each TC event)⁴ leads to greater TC damage costs; this effect is enhanced by a larger increase in TC frequency and/or severity. Moreover, comparing cases 2, 3 and 5, it is found that the cumulative TC damage is more sensitive to the variation in TC severity than TC frequency. In terms of the variation of the cumulative damage costs, Fig. 6(b) shows that the COV associated with case 1 is largest, while that associated with case 4 is smallest. This observation suggests that the nonstationarity in TC process due to the potential impacts of climate change has a significant influence on the variation of the cumulative TC damage. Interestingly, comparing cases 2 and 5, for reference periods up to 55 years, the COV of $D(T)$ associated with case 5 is greater and this observation is inverted for periods exceeding 55 years. This fact indicates that the change of κ_β has a greater effect on the variation of cumulative TC damage than κ_λ at latter stages.

Note that in Fig. 6, the discount rate was assumed to be constant in time. Usually, for service periods of 30 years or less, the future discount rate may be inferred according to the current market interest rates; however, for longer time horizons, a declining discount rate has been widely accepted for the balance between the short-term and long-term concerns and the intergenerational equity [44]. In order to illustrate the impact that the variation of discount rate, $r(t)$, has on the assessment of TC damage costs, we assume that $r(t)$ varies linearly with time, i.e.,

$$r(t) = r_0(1 - \kappa_r t) \quad (41)$$

where κ_r is a parameter indicating the changing rate of $r(t)$; r_0 is the initial discount rate, which is set 3.5%. For case 5 as in Table 1, the

⁴ In the following, the term ‘TC severity’ is used to represent the TC damage costs conditional on the occurrence of one TC event. It is noticed that the TC damage is by nature dependent on both the TC characteristics (e.g., maximum wind speed, rainfall, random track and decay process) and the vulnerability of the area of interested.

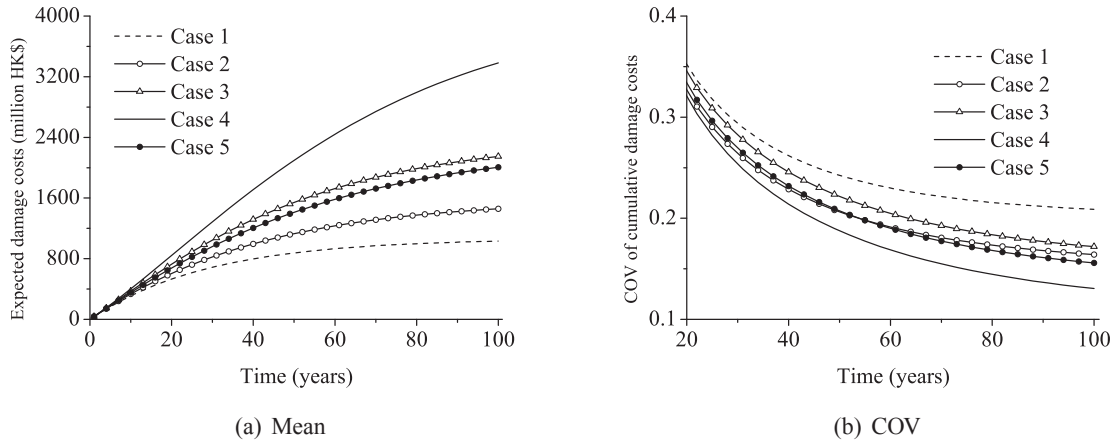


Fig. 6. Statistics of the cumulative TC damage costs for periods up to 100 years for the five cases in Table 1.

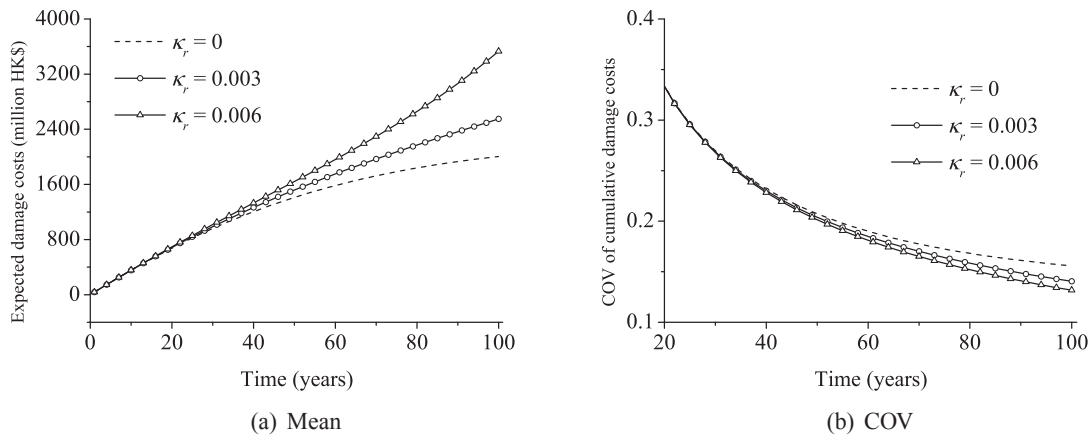


Fig. 7. Statistics of the cumulative TC damage costs for periods up to 100 years for different scenarios of discount rate.

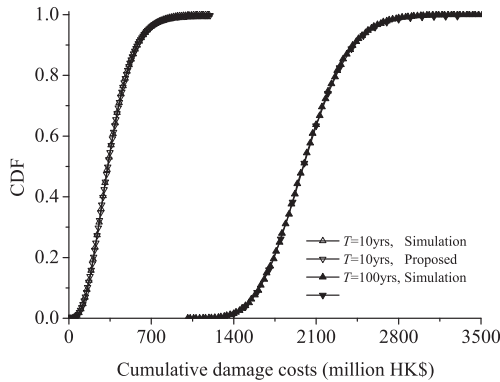


Fig. 8. Comparison between the CDFs associated with the proposed method and those generated from MSC for $T= 10$ and 100 years, respectively.

mean value and COV of cumulative TC damage costs for periods up to 100 years are calculated according to Eqs. (8) and (9) and are plotted in Fig. 7, where κ_r equals 0, 0.003 and 0.006 respectively. It is noted that $r(t)$ drops from 3.5% at initial time to 2.45% at the end of 100 years with $\kappa_r = 0.003$ and drops to 1.4% over 100 years if $\kappa_r = 0.006$. Fig. 7(a) shows that for short-term periods (e.g., less than 40 years), the difference between the damage costs associated with different discount rates is insignificant; however, for wider time horizons, the constant discount rate will inevitably lead to the relatively less value of future damage costs in present terms, while the declining discount rate may better account for the current consideration of the well-beings of future generations. Fig. 7(b) shows that a constant discount rate may lead to an

overestimated variation of TC damage. The above observations imply the relative importance of choosing an appropriate time-variant discount rate to balance the intergenerational equity.

5.3. Probability distribution of cumulative damage costs

Now we consider the probability distribution of cumulative TC damage costs, $F_D(x)$. Firstly, in order to demonstrate the validity of Eq. (31), the CDFs of $D(10)$ and $D(100)$ are obtained by Monte Carlo simulation with one million replications and are compared with the results by Eq. (31). For each simulation run, the procedure of generating a sample of $D(T)$ is as follows.

- Generate the number of TC events, n_T , which follows a Poisson distribution with a mean value of $\int_0^T \lambda(\tau) d\tau$.
- Simulate n_T independent and identically distributed time points, $t_1, t_2, \dots, t_{n_T}$ (in ascending order), with a PDF of $\frac{\lambda(t)}{\int_0^T \lambda(\tau) d\tau}$, $0 < t \leq T$.
- Corresponding to $t_k, k = 1, 2, \dots, n_T$, generate a sample for the TC damage corresponding to $t_k, \tilde{d}(t_k)$, and set $d(t_k) = \frac{\tilde{d}(t_k)}{[1 + r(t_k)]^{t_k}}$ for each k .
- The sum of all $d(t_k)$ is recorded as a sample of $D(T)$.

Fig. 8 plots the simulated histograms of $D(10)$ and $D(100)$ and the calculated CDFs with Eq. (31), where case 5 in Table 1 is considered. The consistency of the simulated and theoretical results demonstrates the accuracy of Eq. (31). Fig. 9 plots the CDFs of $D(10)$ and $D(100)$ associated with the five changing patterns of future TCs as summarized in Table 1. The difference between the CDFs demonstrates that the increase in TC frequency and/or severity leads to larger hurricane

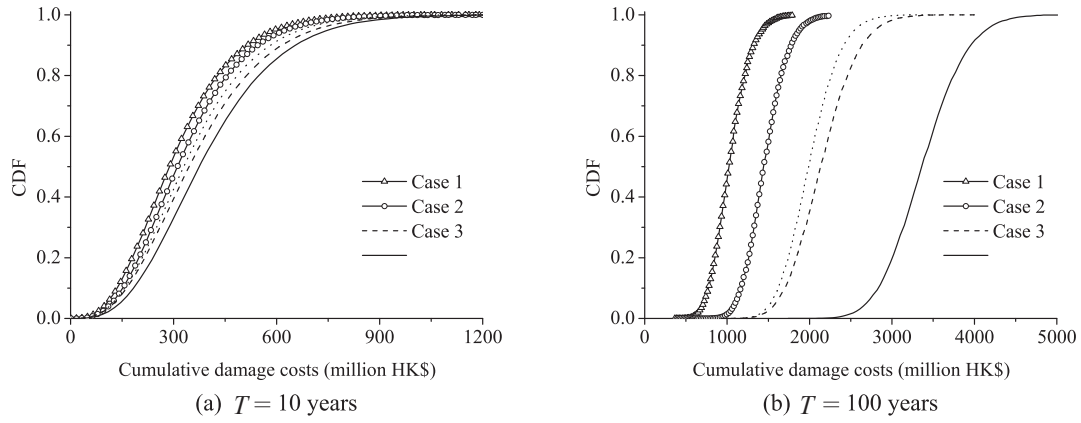


Fig. 9. CDFs of cumulative TC damage costs corresponding to the five cases in Table 1.

damage costs, which is consistent with the observations from Fig. 6. From the probability distributions in Fig. 9, one can make some confidence statements on the estimate of TC damage costs, which can serve as a performance metric in the context of probability-based decision-making. For example, with Fig. 9(b), one may conclude that 'I'm 90% confident that the cumulative TC damage costs do not exceed 2.42 billion HK dollars for a subsequent period of 100 years if both TC frequency and severity vary in time as predicted from historical data (i.e., case 5 in Table 1)'. Moreover, the probability distribution of $D(T)$ also provides a quantitative description of the upper tail behaviour of the cumulative TC costs in terms of exceeding probability, which further gives the characteristic values of damage costs (e.g., the 90th percentile). Such information is essentially important in risk-informed community safety estimate and enhancement. For instance, for a reference period of 100 years, the characteristic values of TC damage costs with exceeding probability of 10% are estimated as 1.33, 2.64 and 2.42 billion HK dollars respectively for cases 1, 3 and 5 as summarized in Table 1. The difference between these characteristic values is more significant for a longer reference period, as observed from the comparison between Fig. 9(a) and (b).

6. Conclusions

A probability-based approach has been proposed in this paper to estimate the future TC damage costs for coastal areas, which also enables the impacts of climate change on TC damage costs to be incorporated. The following conclusions can be made from this paper.

- (1) The proposed approach estimates both the moments (mean value and variance) and the probability of cumulative TC damage costs in an explicit form, which takes into account the time-variation of future TC frequency and TC severity (reflected by the TC damage

costs triggered by the TC event). The applicability of the proposed method is demonstrated through an application to the future TC damage assessment of Hong Kong, China – a TC-prone area. The accuracy of the proposed approach is verified through an comparison with the results obtain from Monte Carlo simulation.

- (2) The increase in TC severity and/or frequency leads to greater hurricane damage costs. The cumulative TC damage is more sensitive to the variation in TC severity than that in TC frequency, indicating the relative importance of predicting the changing pattern of future TC damage costs associated with each TC event. Moreover, taking into account the balance of intergenerational equity, a declining discount rate rather than a constant one may better account for the current consideration of the well-beings of future generation, especially for long-term reference periods.
- (3) The selection/prediction of different changing patterns of future TC severity and/or frequency affects the probability distribution of cumulative TC damage costs significantly, which further determines the upper tail behaviour of damage costs and the characteristic values with a certain confidence level. Presenting the probability-based behaviour of TC damage costs employing the proposed method appears to be an important step toward risk-informed decision-making for TC-prone areas.

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Appendix A. On the convergence of $F_D(x)$

To prove the convergence of $F_D(x)$, we first have the following lemma for the convergence test of integrals.

Lemma 1. Let $a \in \mathbb{R}$ and $f: [a, \infty) \rightarrow \mathbb{R}$ be a monotonic and differentiable function such that $f(x) \rightarrow 0$ when $x \rightarrow \infty$ and f' is integrable on $[a, x]$ for $\forall x \geq a$. If g is a continuous function on every subinterval $[a, x] \subset [a, \infty)$ and $\int_a^x g(x)dx$ is bounded, then $\int_a^\infty f(x)g(x)dx$ is convergent for $\forall \tau \in \mathbb{R}$.

Proof. See, e.g., [50] (p.p. 442). $\square$

Now, with Lemma 1, we show that the integral in Eq. (31) is convergent for $\delta(\tau)$ and $\theta(\tau)$ in Eqs. (37) and (38). Let $h(\tau) = \frac{\exp(\delta(\tau))}{\tau}$. Since $1 + \left(\frac{\tau \beta(t)}{[1 + r(t)]^t}\right)^2 \rightarrow \infty$ as $\tau \rightarrow \infty$, $\lim_{\tau \rightarrow \infty} \delta(\tau) \rightarrow -\int_0^T \lambda(t)dt$. Thus, $\lim_{\tau \rightarrow \infty} h(\tau) \rightarrow 0$. Moreover, the integral $\int_0^a \sin \theta_0(\tau) - \sin(\theta_0(\tau) - x\tau) d\tau$ is bounded for an arbitrary positive number a . By further noting the monotonicity of $h(\tau)$ as τ is large enough, the convergence of F_D is proven.

Appendix B. On the simplification of $\delta(\tau)$ and $\theta(\tau)$ assuming a normal distribution for TC damage costs associated with each TC event

We first note that if a random variable X follows a normal distribution with a mean value of μ_X and a standard deviation of σ_X , the moment

generating function of X is $\phi_X(\tau) = \exp\left(\mu_X\tau + \frac{1}{2}\sigma_X^2\tau^2\right)$. Assume that $X \geq 0$ (i.e., the probability that $X < 0$ is negligible). With an arbitrary positive constant k ,

$$\int_0^\infty \cos(kx) \cdot f_X(x) dx + i \int_0^\infty \sin(kx) \cdot f_X(x) dx = \phi_{iX}(ik) = \exp\left(ik\mu_X - \frac{1}{2}\sigma_X^2 k^2\right) = \exp\left(-\frac{1}{2}\sigma_X^2 k^2\right) \cos(k\mu_X) + i \exp\left(-\frac{1}{2}\sigma_X^2 k^2\right) \sin(k\mu_X) \quad (\text{B.1})$$

As a result, one has

$$\int_0^\infty \cos(kx) \cdot f_X(x) dx = \exp\left(-\frac{1}{2}\sigma_X^2 k^2\right) \cos(k\mu_X) \quad (\text{B.2})$$

and

$$\int_0^\infty \sin(kx) \cdot f_X(x) dx = \exp\left(-\frac{1}{2}\sigma_X^2 k^2\right) \sin(k\mu_X) \quad (\text{B.3})$$

Thus, the integral $\int_0^\infty \cos\left(\frac{z\tau}{(1+r(t))^t}\right) \cdot f_0(z, t) dz$ in $\delta(\tau)$, as well as the integral $\int_0^\infty \sin\left(\frac{z\tau}{(1+r(t))^t}\right) \cdot f_0(z, t) dz$ in $\theta_0(\tau)$ can be simplified with Eqs. (B.2) and (B.3) by setting $k = \frac{\tau}{(1+r(t))^t}$.

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